The Impact of Consumption Hassle on Pricing Schedules

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*Corresponding author. Financial support from the General Research Fund of the School of Business, University of Kansas, is gratefully acknowledged.
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Abstract

A curious phenomenon in the retail pricing of many product categories (e.g., tuna, frozen orange juice, tomato paste) is the existence of quantity surcharges. A related curiosity is the existence of both discounts and surcharges within the same product category. To explain the former phenomenon, previous researchers invoke heterogeneity in consumption parameters (Gerstner and Hess 1987), heterogeneity in search costs (Binkley and Bejnarowicz 2003), or concern for retail price image (Sprott, Manning, and Miyazaki 2003). To the best of our knowledge, there is no extant explanation for the latter phenomenon. We add to this rich stream of work by proposing a novel explanation for quantity surcharges. Our explanation is based on the notion of consumption hassle. We analyze a market that is heterogeneous in a hedonic parameter that influences valuations as well as the effective cost of the consumption hassle. We then derive consumer choices (small pack, large pack, or two small packs) taking into account Individual Rationality (IR) and Incentive Compatibility (IC) constraints. In the absence of consumption hassle, we obtain two segments with one purchasing the small pack and the other purchasing the large pack. Moreover, the optimal pricing for the seller involves quantity discounts. However, with the introduction of consumption hassle, the market potentially splits into three segments: one purchasing the small pack, another purchasing two small packs, and the third purchasing the large pack. Moreover, the optimal pricing for the seller involves quantity surcharges. Overall, our analytical findings offer an additional explanation for the phenomenon of quantity surcharges. More importantly, they offer a rationale for the existence of multiple pricing schedules within the same product category by explicitly recognizing variations in consumption hassle.

Key words: Quantity Surcharges, Retail Pricing, Consumption Hassle
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INTRODUCTION

Motivation and Research Objectives

A curious phenomenon in the retail pricing of many product categories (e.g., tuna, frozen orange juice, tomato paste) is the existence of quantity surcharges. Kaufman, MacDonald, Lutz and Smallwood (1997) report that the 12 oz. size of tuna is, on average, 1.41 times as costly as two 6 oz. cans. Cude and Walker (1984) document surcharges in the category of frozen orange juice and canned tuna. Similarly, Nason and Della Bitta (1983) report surcharges in tomato paste and canned tuna. Clearly, some product categories are marked by significant and persistent incidences of surcharging.1

An even more surprising curiosity is the existence of both discounts and surcharges within the same product category. Consider, for example, the tuna category. Currently, tuna is offered in two formats: cans and pouches. Examining pricing schedules across fifteen retail outlets in a large metropolitan area in the northwest region of the U.S., we find surcharges in the can format but discounts in the pouch format. Specifically, the average surcharge among can brand-pairs is 26.21% whereas the average discount among pouch brand-pairs is 27.84%. Both surcharges and discounts are statistically different from zero (t = 7.26 and t=-12.76, n = 59 and 15, respectively).2

1 We also find surcharging for tuna and tomato paste in an online store, netgrocer.com. In the case of tuna, the 6 oz. size of Bumble Bee (solid white albacore in water) retails for $1.95 whereas the 12 oz. size retails for $4.58 – a surcharge of 17%. Similarly, the 6 oz. size of Contadina tomato paste retails for $0.79 whereas the 12 oz. size retails for $1.95 – a surcharge of 23%.

2 A similar curiosity emerges within the orange juice category. We find greater incidence of surcharges in the frozen form compared to the liquid form, which always exhibits discounts.
Given these observations, our primary research objective in this paper is to develop and analyze an analytical model that can potentially explain these phenomena. In addition, we also wish to delineate insights that can assist practitioners in the design of optimal pricing schedules. In contrast to the extant explanations found in the literature, our explanation is based on the notion of hassle associated with the consumption process (consumption hassle).

Since consumption hassle is a key ingredient of our analysis, we pause to say a few words about how these may arise in a typical consumption situation. In the case of tuna, consumers opening a can of tuna have to exert some care to extract the product and dispose the liquid material. (Typically, the desired solid product is immersed in some kind of liquid, usually oil or water). Moreover, the odorous nature of the product necessitates caution to ensure that all of the liquid is carefully disposed in the sink without spilling. Similarly, in the case of frozen orange juice, the consumption hassle consists of extracting the concentrate out of the package, transferring it to the juice pitcher, adding the correct amount of water, and stirring to ensure that the concentrate is thoroughly dissolved. Finally, in the case of tomato paste, the consumption hassle consists of completely extracting the sticky substance out of the can.

Model Features and Central Insights

The key features of our analytical work are as follows. We consider a market in which consumers are heterogeneous with respect to an underlying hedonic trait, \( \theta \). This underlying hedonic trait influences the structure of preferences. In particular, the hedonic trait influences the valuation for the product as well as the distaste for the consumption
hassle. As specified in Chiang and Spatt (1982) and highlighted in Tirole (1988, p. 151), consumers who have the highest valuation for the product are also those with the highest distaste for the consumption hassle. Finally, as in Tirole (1998, p. 159) and much of the economics literature, consumers exhibit diminishing marginal utility for the second unit of the product. To keep the analysis tightly focused on the issue of consumption hassle, we assume that the small pack contains one unit of the product whereas the large pack contains two units of the product. Lastly, the seller chooses prices for the two pack sizes with the objective of maximizing profits.

In this context, we formally demonstrate that surcharging can emerge as a profit-maximizing strategy. Further, surcharges arise in two manifestations: two-segment and three-segment. In some regions of the parameter space, only two adjacent segments emerge: one consuming the small pack and the other consuming the surcharged, large pack. In other regions of the parameter space, three adjacent segments emerge: one consuming the small pack, another consuming two small packs, and a third consuming the surcharged, large pack. The basic intuition for surcharging is that substituting two small packs for one large pack incurs the consumption hassle of opening an additional pack; consequently, the seller is able to increase profits by surcharging the convenient, large pack. What makes surcharging feasible is the structure of preferences. Consumers with high values of the hedonic trait maximize their utility when they consume the surcharged large pack. In contrast, consumers with low values of the hedonic trait maximize their utility when they consume the small pack.

Overall, our incremental insight is that surcharging is simply a form of segmented pricing with consumption hassle ensuring the necessary separation between consumers.
If the separation is perfect, we obtain the aforementioned two-segment solution. On the other hand, if there is some leakage, we obtain the aforementioned three-segment solution.

Managerially, these findings offer a novel perspective on the incidence of surcharging among products that are characterized by consumption hassle. As such, they are an aid towards the design of optimal pricing schedules. From an academic perspective, our work echoes the findings documented in the literature on quality premia, wherein successive levels of quality are priced disproportionately higher (see, for example, Verboven 1999).

**Extant Literature on Surcharges**

Of course, the issue of quantity surcharges has been investigated in previous work. Gerstner and Hess (1987) analyze a model in which consumers are fully informed about prices but are heterogeneous in terms of consumption rates, storage costs, and the transaction costs of going to the store. They obtain the general finding that these three sources of heterogeneity can result in a variety of package sizes with unit prices that may reflect quantity surcharges or discounts. In effect, the choice of package size and prices (surcharge or discounts) are the key outputs of their analysis.

Our work extends their contribution by introducing the notion of consumption hassle, a construct markedly different from the transaction cost of going to the store. *This is because while all products incur the transaction costs of going to the store, only some*
products subject consumers to consumption hassle. For example, frozen orange juice imposes significant consumption hassle whereas liquid orange juice is readily opened and consumed. Thus, while both product formats impose a transaction cost of going to the store only the frozen format imposes consumption hassle.

Two other explanations for quantity surcharges can be found in the literature. The first explanation hinges on the notion of search costs. Specifically, high search costs on the part of some consumers preclude careful price comparisons. As such, these buyers fail to select the best buy. This is the viewpoint espoused in Binkley and Bejnarowicz (2003) in their analysis of quantity surcharges in tuna. Similarly, Clerides and Courty (2010) suggest that consumer inattention can cause quantity surcharges in the case of laundry detergents during promoted periods.

The second explanation proposes that in those instances where the small size is also the most popular size, the retailer’s desire to establish a low-price image puts downward pressure on the price of small size because it can be found in the shopping basket of many consumers. This reduction in price for the small size without a corresponding decrease in the price of the large size implicitly induces a quantity surcharge. This is the viewpoint espoused in Sprott, Manning, and Miyazaki (2003).

We acknowledge that these explanations are crucially important but suggest that there is more to the story. In Binkley and Bejnarowicz (2003), for example, the central thesis is that buyers with high search costs find it too costly to obtain full information about prices. Similarly, Clerides and Courty (2010) propose consumer inattention. However, their work leaves open the following questions: Why does surcharging exist

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3 Some researchers even find it appropriate to set the transaction costs of going to the store as zero (see, for example, Jeuland and Narasimhan 1985). In their view, shopping cost is a sunk cost that must be incurred
even in online channels where the unit price is clearly displayed? Moreover, why can’t brands in other categories take similar advantage of consumer search costs – why are surcharges systematically restricted to certain product categories?

Similarly, in Sprott, Manning and Miyazaki (2003), the central thesis is that retailers concerned with a low-price image come to offer discounts on popular products. However, their work leaves open the following question: Why do rational consumers inefficiently purchase the large pack? Why don’t they simply buy two packs of the small size and avoid the surcharge?

Finally, none of the extant explanations can reconcile the existence of both discounts and surcharges within the same category. As mentioned previously, a unique feature of the tuna category is that the product is offered in two formats: the traditional can format and a new pouch format. It is generally believed that the value-added of the pouch format is that it is relatively easy to open and there is no need to drain any filler liquid. Compared to the can format, the pouch format thus reduces or eliminates consumption hassle. Notably, our theory can accommodate surcharges or discounts within the same product category by explicitly recognizing such variations in consumption hassle. Our work also complements recent empirical evidence from the tuna category that demonstrates that two six-ounce cans of tuna may not be equivalent to one, twelve-ounce can (Chouinard, McCluskey, and Sprott 2004). In effect, they find that these products are imperfect substitutes thereby supporting our proposed conceptualization of consumption hassle.

Overall, we note that there is considerable interest in the topic of surcharges and there are many well-established explanations for the observed phenomena. We add to each week.
this body of work by formally introducing and analyzing the notion of consumption hassle. *We hope that our research paints an additional layer of understanding on the works of Chouinard, McCluskey, and Sprott (2004), Clerides and Courty (2010), Binkley and Bejnrowicz (2003), Gerstner and Hess (1987), and Sprott, Manning and Miyazaki (2003).*

The rest of the paper is organized in the following manner. We first present our model and assumptions. We then describe a few important properties of the market that we analyze. Next, we illustrate the optimality of surcharging and characterize prices. We then offer some empirical evidence for our model assumptions. We conclude with implications for marketers and policy-makers.

**MODEL AND ANALYSIS**

**Basic Model Structure**

We begin by presenting the essential assumptions employed in our analytical work.

**A1: Market and Preferences**

We consider a heterogeneous market wherein consumers differ on a hedonic trait, $\theta$. In our market, $\theta$ is distributed uniformly over the interval $[0, 1]$. As suggested previously, $\theta$ influences the valuation for the product as well as the distaste for the consumption hassle. Specifically, consumers who enjoy the product the most are also those that impute a high cost to the consumption hassle (Chiang and Spatt 1982, Tirole
The intuition is that the hedonic trait influences both the enjoyment of the product (and therefore, valuation or willingness-to-pay) as well as the effective cost incurred due to the consumption hassle.

Indeed, specifying a relationship between valuation and the effective cost of some undesirable action is a fairly accepted conceptualization in the marketing literature. Jeuland and Narasimhan (1985) assume a positive correlation between valuation (consumption rate) and the cost of inventorizing a product. Gerstner and Holthausen (1986) specify that high-valuation customers will have a higher opportunity cost of time than the low-valuation customers; consequently, a given level of consumption hassle is more onerous for high-valuation customers. Gerstner and Holthausen offer the following rationale for this specification, namely “positive correlation between income and willingness to pay and between income and value of time.” Finally, we note that such associations have been conceptualized in other domains as well. For example, Desiraju and Shugan (1999) assume correlation between consumer valuation and time of arrival in the market.

Following prior literature, we also specify a positive association between valuation and the effective cost of the consumption hassle. Consistent with Chiang and Spatt (1982), we specify that valuation increases in a concave manner with respect to θ whereas the cost of the consumption hassle increases linearly with respect to θ. We thus express valuation via the function \( v(\theta) \) with \( v'(\theta) > 0, \ v''(\theta) < 0 \), normalizing \( v(0) = 0 \). We express the cost associated with consumption hassle as \( h(\theta) = \alpha \theta \), where \( \alpha \) is simply a scaling parameter that reflects the degree of consumption hassle. Of course, we hasten
to note that our main finding pertaining to the emergence of surcharges is robust to the specific functional form assumed for $v(\theta)$.

Finally, as in Tirole (1988, p. 159) and in much of the economics literature, we specify that the valuation for the large pack exhibits diminishing utility. Specifically, consumer valuation for the large pack relative to the small pack is captured via the parameter $K$, where $1 < K < 2$. This interval for $K$ ensures diminishing utility for the second unit of the product.

Notice that a consumer who values the small pack at $v(\theta)$ will value the large pack at $Kv(\theta)$. Relating the valuations of the small pack and the large pack in this manner yields two appealing properties. First, consumers who value the small pack highly are also those that value the large pack highly. Second, consumers who value the small pack highly are those who are willing to pay more for the second unit since this valuation is given as $(K - 1) v(\theta)$.

A2: Marginal Costs

We assume that the seller incurs marginal cost, $c$, for the small pack and $2c$ for the large pack. It is possible that marginal costs will actually be lower than $2c$ for the large size; however, this will only put downward pressure on the price of the large pack. As such, differences in costs cannot explain surcharges and we are content to assume costs of $c$ and $2c$. However, in sensitivity analysis, we will also demonstrate that our main result of surcharges emerges even with departures from this assumption.
A3: Information Structure

We assume that the distribution of the hedonic parameter, $\theta$, prices, the parameter $K$, and the structure of hassle costs are common knowledge. We further assume that each consumer is aware of his (or her) hedonic parameter, $\theta$.

A4: Objective Functions

There are three purchase options for the consumer: purchase the small pack (1), purchase the large pack (2), or purchase two small packs (1+1). Consumer utility, $U$, for each of the three purchase options can now be written as:

\[
\begin{align*}
U_1 &= v(\theta) - \alpha \theta - p \\
U_{1+1} &= K \cdot v(\theta) - 2 \alpha \theta - 2 p \\
U_2 &= K \cdot v(\theta) - \alpha \theta - P
\end{align*}
\]

In these equations, $p$ and $P$ represent the price of the small pack and large pack, respectively. Of course, the consumer could choose not to purchase ($\phi$). The additive expression for utility comprises of an intrinsic preference term, product attributes (consumption hassle), and price. Such a formulation is used quite commonly in logit models of choice (see, for example, equation (1) in Basu, Mazumdar, and Raj (2007)). We define $IR_1$ as the individual rationality condition for the purchase of the small pack over no purchase. Similar definitions hold for $IR_2$ and $IR_{1+1}$. We define $IC_{2,1}$ as the Incentive Compatibility condition by which a consumer prefers consumption choice 2.
over 1. Similar definitions hold for IC_{2,1+1} and IC_{1+1,1}. The objective of the consumer is to employ these IR and IC conditions to choose the option with the highest non-negative utility. Obviously, her preferences (θ) and observed prices, p and P, play a key role in determining the chosen option.

Finally, we consider a monopoly seller choosing prices for the two packs with a view to maximize single-period profits.  

**Initial Analysis**

We normalize the number of consumers in the market to 1. Before we present our solutions, we derive some general properties of the market.

We first note that the rate of change of the three utility expressions (w.r.t. θ) can be written as:

\[ U_1' = v'(θ) - α \]
\[ U_{1+1}' = K. v'(θ) - 2α \]
\[ U_2' = K. v'(θ) - α \]

We see that the slope of U_2 is greater than either the slope of U_1 or U_{1+1}. The slope of U_{1+1} may or may not be greater than the slope of U_1 - it depends on the values of K and α. Using equation (2), we see that U_1 intersects the x-axis at −p whereas U_{1+1} intersects the x-axis at −2p. The utility U_2 intersects the x-axis at −P, which is below −2p for

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4 In effect, we abstract away from issues of competition to focus exclusively on the impact of consumption hassle. This is consistent with the prescriptions found in Shugan (2003).
surcharges (please see Figure 1). We are now in a position to state the first property of our market:

**Property 1:** If the pricing schedule exhibits quantity discounts, consumers never choose the 1+1 option.

In this event, \( U_2 \) begins above \( U_{1+1} \) when \( \theta = 0 \). Moreover, since its slope is greater, \( U_2 \) dominates \( U_{1+1} \) everywhere. Thus, surcharges are a necessary condition for the existence of the 1+1 segment. This is also understood in straightforward fashion by noting that consumers will not endure the higher consumption hassle of the 1+1 option if purchase of 2 can be made with a discount. However, while surcharges are a necessary condition for the existence of the 1+1 segment, they are by no means a sufficient condition. That is, even with quantity surcharges, the values of \( K \) and \( \alpha \) could be such that \( U_{1+1} \) never overtakes \( U_1 \) (or at least before \( U_2 \) overtakes \( U_1 \)). As such, consumers may not choose the 1+1 option even in the presence of surcharges.

**Property 2:** If the overall market includes purchasers of 1 and 1+1, the market always begins with the segment that purchases 1.

Property 2 states that when the market includes purchasers of 1 and 1+1, individuals who purchase 1 are characterized by lower values of \( \theta \) as compared to individuals who purchase 1+1. To see this, let IR\(_1\) just become positive at \( \theta^* \). At this point, \( v(\theta^*) - \alpha\theta^* = p \). Substituting for \( p \), we obtain \( U_{1+1} = (K - 2) \). \( v(\theta^*) \), which by
definition, is negative. We thus have to move a little to the right before the market for 1+1 can potentially begin.

Property 3: If the overall market includes purchasers of 1 and 2, the segment purchasing 1 always lies to the left of the segment purchasing 2. This is true irrespective of whether the pricing structure exhibits discounts or surcharges.

Let the dividing point be $\theta^*$. The condition for 2 to dominate 1 is $(K - 1) v(\theta) > P - p$. If equality is achieved at $\theta^*$, the inequality holds at all points to the right of $\theta^*$ since $v'(\theta) > 0$.

Property 4: If the overall market consists of purchasers of 1+1 and 2, the segment purchasing 1+1 always lies to the left of the segment purchasing 2.

Here, let the dividing point be $\theta^{**}$. The condition for 2 to dominate 1+1 is $\alpha \theta > P - 2 p$. If equality is achieved at $\theta^{**}$, the inequality holds at all points to the right of $\theta^{**}$ since $\alpha \theta$ is increasing in $\theta$.

The aforementioned results reveal that if the overall market consists of purchasers of 1, 1+1, and 2, the market consists of adjacent segments in the order: $[\phi, 1, 1+1, 2]$.  

5 Strictly speaking, the market could also look like $[\phi, 1, 1+1, 1, 2]$, but we conjecture that this is unlikely to arise. In our main analysis, the use of a specific functional form for $v(\theta)$ allows us to analytically rule out this possibility. This is formally described in Appendix A.
If the market only consists of purchasers of 1 and 2, it consists of adjacent segments in the order: \([\phi, 1, 2]\). The delineation of these properties greatly facilitates our subsequent exposition and analysis.

**Main Analysis**

The analysis consists of the following. Given a pair of prices, \(p\) and \(P\), consumers make their choices employing the \(\text{IR}(\theta)\) and \(\text{IC}(\theta)\) constraints. This results in segments buying nothing, 1, 1+1, or 2. The seller’s objective function is obtained by taking the width of the segments for 1, 1+1, and 2, and then multiplying those widths with the appropriate margins (\(p – c\), 2.[\(p – c\)], and \(P – 2c\), respectively). Obviously, the seller seeks those prices \(p\) and \(P\) that maximize the objective function. Then, comparison of \(P\) and \(2p\) reveals whether the optimal pricing schedule reflects discounts or surcharges.

In order to obtain expressions for the widths of the segments, we need to specify a functional form for \(v(\theta)\). With a view to obtain analytical results as far as possible, we thus assume:

\[ A5: \quad v(\theta) = \theta^{1/2}. \]

Our analysis proceeds in two stages. We first analyze a benchmark model with no consumption hassle. Then, we include consumption hassle and demonstrate the emergence of surcharges and the characterization of prices.
Analysis Excluding Consumption Hassle (Benchmark)

Here, the consumer has the following possible actions at his (or her) disposal:

a. Purchase nothing
b. Purchase the small pack
c. Purchase the large pack

The seller is unable to charge a premium for the large pack because the absence of consumption hassle implies that there are no incremental costs of substituting two small packs for one large pack. Specifically, if the large size is surcharged, the consumer would simply purchase two units of the small pack and the seller would earn zero profits on it. Recognizing this, surcharges do not arise and the possibility of purchasing two small packs is therefore not considered. The conditions for purchasing the small and large pack are given as:

\[ \theta^{1/2} - p > 0 \quad \Rightarrow \quad \theta > p^2 \quad \text{(3a)} \]

\[ K \theta^{1/2} - p > \theta^{1/2} - p \quad \Rightarrow \quad \theta > \left( \frac{p - p}{K - 1} \right)^2 \quad \text{(3b)} \]

Equation (3a) is the individual rationality constraint for purchase of the small pack to occur. Equation (3b) is the incentive compatibility constraint to depict the consumer’s preference for the large pack over the small pack. Given these constraints, the market divides as follows: consumers in the interval \([ p^2, \left( \frac{p - p}{K - 1} \right)^2 ]\) purchase the small pack.
whereas consumers in the interval \([\left(\frac{P-p}{K-1}\right)^2, 1\]\) purchase the large pack. The profits for the seller can now be written as:

\[
\pi = (p - c) \left[ \left(\frac{P-p}{K-1}\right)^2 - p^2 \right] + (P - 2c) \left[ 1 - \left(\frac{P-p}{K-1}\right)^2 \right]
\]  

(4)

Differentiating equation (4) with respect to \(p\) and \(P\) and after some algebra, we are able to obtain the following prices and the market partitions:

\[
p = \frac{c + \sqrt{c^2 + 3}}{3}
\]  

(5a)

\[
P = p + \frac{c + \sqrt{c^2 + 3(K-1)^2}}{3}
\]  

(5b)

Market for small pack:

\[
\left[ \frac{2c^2 + 3 + 2c\sqrt{c^2 + 3}}{9}, \left(\frac{c + \sqrt{c^2 + 3(K-1)^2}}{3(K-1)}\right)^2 \right]
\]  

(5c)

Market for large pack:

\[
\left[ \left(\frac{c + \sqrt{c^2 + 3(K-1)^2}}{3(K-1)}\right)^2, 1\right]
\]  

(5d)

Equations (5a) and (5b) reveal that the price of the small pack is independent of \(K\) whereas the price of the large pack is increasing in \(K\). More importantly, since the second term in equation (5b) is less than \(p\), the optimal price for the large pack is less
than twice the small pack. Finally, as $K$ approaches 2, the market for the small pack shrinks. When $K = 2$, it disappears altogether and the seller only offers the large pack.

In summary, our benchmark model obtains the expected finding that the seller responds to diminishing utility by offering a quantity discount. As such, this structure serves nicely to examine how the optimality of quantity discounts is altered by the inclusion of consumption hassle.

**Analysis Incorporating Consumption Hassle**

Here, we present our findings regarding the optimal pricing schedule in the face of consumption hassle. The profit function for the seller is:

\[
\pi = (M_1)(p-c) + (M_{1+1})(2(p-c)) + (M_2)(P-2c), \tag{6}
\]

where $M_1$, $M_{1+1}$, and $M_2$ denote the widths for the 1, 1+1, and 2 segments, respectively. Here, unlike the case of no consumption hassle, it is also possible for a 1+1 segment to emerge; however, if this does not arise, $M_{1+1}$ is zero. The seller maximizes the profits in (6) while explicitly recognizing that each pair of prices $[p, P]$ generates a different market partition (widths of $M_1$, $M_{1+1}$, and $M_2$).

We first numerically investigate the optimal pricing schedule in the $K-\alpha$ parameter space. Fortunately, within our model structure, $K$ and $\alpha$ have defined bounds so we need not fear excluding any region of the parameter space (These bounds are derived as follows: the incremental value of the second unit is $K \cdot v(\theta) - v(\theta)$ and this must exceed the cost of production, $c$, for at least the highest valuation customer with
\( v(\theta) = 1; \) consequently, \( K - 1 > c \Rightarrow K > 1 + c \) else it is not worthwhile to sell the large pack and the question of discounts / surcharges is moot. In addition, \( K < 2. \) If \( K = 2, \) the seller only offers the large pack and again the question of discounts / surcharges is moot. With respect to the parameter \( \alpha, \) we expect hassle to be non-negative, hence, \( \alpha \geq 0. \) Finally, \( \alpha \leq 1, \) else the consumer with the highest valuation will find the small pack unattractive at any positive price – an unappealing feature). We consider a grid of 10,000 (100X100) points in the \( K-\alpha \) space. At each point, a given price pair generates a specific market partition and the corresponding profit can be computed. We then search for the price pair that generates the highest profit and also note the following two features: (i) whether the pricing schedule involves surcharges or discounts, and (ii) the number of segments that market splits into. We obtain:

**Proposition 1:** Under Assumptions A1 – A5, the parameter space divides into the following three regions (please see Figure 2):

- **D2:** Optimal pricing schedule is quantity discounts and the market splits into two segments (1, 2)
- **S2:** Optimal pricing schedule is quantity surcharges and the market splits into two segments (1, 2).
- **S3:** Optimal pricing schedule is quantity surcharges and the market splits into three segments (1, 1+1, 2).
Figure 2 numerically demonstrates the existence of regions in the parameter space where it is profit-maximizing for the seller to employ quantity surcharges. In effect, the seller obtains extra surplus from consumers at the upper end of the market who buy the surcharged, large pack. In our model, hassle costs ensure the necessary separation. If this separation is perfect, we obtain the two-segment solution. If the separation induces some leakage, we obtain the three-segment solution.

Since we expect real-world markets to exhibit all types of purchasing (1, 1+1, and 2), we focus on characterizing prices in the S3 region. We have:

**Proposition 2a:** The surcharge and comparative statics in region S3 are described as follows:

(i) \( P - 2p = \frac{\alpha}{2} \)

(ii) \( \frac{\partial P}{\partial K} > 0, \frac{\partial P}{\partial \alpha} > 0, \frac{\partial P}{\partial \alpha} < 0, \) and \( \frac{\partial P}{\partial \alpha} < 0, \)

For completeness, we also characterize prices in the D2 and S2 regions. We have:

**Proposition 2b:** In regions D2 & S2:

(i) \( P - p = \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right) \),

(ii) \( 1 + \frac{p-c}{\alpha} + \left( \frac{1-6ap + 2ac - (1-2ap)\sqrt{1-4ap}}{2\alpha^2\sqrt{1-4ap}} \right) = 0 \),
is the condition for region S2 to emerge, and

(iv) \( \frac{\partial P}{\partial K} > 0, \frac{\partial P}{\partial K} = 0, \frac{\partial P}{\partial \alpha} < 0, \) and \( \frac{\partial P}{\partial \alpha} < 0, \)

**Proof:** See Appendix B.

*Intuition for Key Findings in Propositions 1 & 2*

We first outline the intuition for the three regions displayed in Figure 2. Broadly, surcharging is feasible as long as the sum of \( K + \alpha > \) constant (this is the approximate shape of the curve described in Proposition 2b (iii), the boundary between D2 and S2). First consider the parameter \( K \). Clearly, \( K \) facilitates surcharging – if the second unit does not provide much value, the optimal pricing schedule is forced towards quantity discounts. Next consider the parameter \( \alpha \). This parameter helps prevent leakage. Absent high values for \( \alpha \), consumers will simply avoid surcharges by consuming two units of the small pack. For these reasons, both \( K \) and \( \alpha \) facilitate surcharging and roughly behave in a compensatory fashion.

Next, we provide some intuition for the comparative statics. The intuition for why \( p \) depends on \( K \) in the three segment solution \( \left( \frac{\partial P}{\partial K} > 0 \right) \) but is independent of \( K \) in the two segment solution \( \left( \frac{\partial P}{\partial K} = 0 \right) \) is as follows. In the three segment solution, the 1+1 segment purchases two small packs. This makes the price of the small pack sensitive to
the marginal utility of the product (the parameter K). In contrast, there is no such consideration in a two segment scheme where the consumers who buy the small pack are not influenced by the parameter K. Thus, K has no impact on the price of the small pack in the two segment solution. The intuition for \( \frac{\partial P}{\partial \alpha} > 0 \) and \( \frac{\partial p}{\partial \alpha} < 0 \) is straightforward: increases in hassle degrade consumer utility and thereby limit pricing power. Moreover, the latter comparative static is greater in magnitude because consumption hassle looms large relative to the lower price of the small pack; consequently, its impact is more pronounced.

Finally, we discuss the rationale for the existence of the S3 region vis-à-vis the S2 regions. Note that across the regions S2 and S3, \( \frac{\partial P}{\partial K} > 0 \). Now, consider a particular value of \( \alpha \). The price of the big pack, P, increases as we move from left to right across regions S2 and S3 (increasing K). The price of the small pack remains unchanged in S2 since \( \frac{\partial p}{\partial K} = 0 \). Consequently, at some value of K, the surcharge is high enough that it is optimal for some consumers to purchase the large quantity, albeit in the form of two small packs. The switch to S3 also occurs at a lower value of K for a higher value of \( \alpha \). This is because although \( \frac{\partial P}{\partial \alpha} < 0 \) and \( \frac{\partial p}{\partial \alpha} < 0 \), the latter comparative static is greater in magnitude. Consequently, as \( \alpha \) increases, the difference between P and 2p also increases and the S2-S3 boundary slopes up and to the left.

Overall, our analysis demonstrates the optimality of surcharging in the presence of consumption hassle. However, it is noteworthy to recognize that the incorporation of
consumption hassle does not preclude discounts. Even within our model, discounts can appear in those product categories whenever $K$ and/or $\alpha$ are low.

**Sensitivity Analysis**

It is natural to inquire if surcharging will emerge as the optimal pricing schedule when other functional forms are chosen for $v(\theta)$. It is also natural to inquire if surcharging will emerge as the optimal pricing schedule when there is a cost advantage in producing the large pack. Using numerical analysis, we are able to find the existence of surcharges even for $v(\theta) = \theta^{3/5}$ or $\theta^{3/4}$. Similarly, we find that surcharges continue to emerge when costs are specified as $c$ and $1.8c$ for the two pack sizes.

**EMPIRICAL EVIDENCE FOR MODEL ASSUMPTIONS**

The purpose of this section is to provide some evidence for our model assumptions. Accordingly, we recruited a sample of 50 undergraduate students to participate in two simple paper and pencil surveys.

For the first survey, subjects were shown pictures of both the canned version of tuna as well as the new, pouch format. Both products were of the same brand and we clearly stated that both formats contained the same amount of tuna. We then asked them the maximum they were willing to pay for either product. We found that students were willing to pay substantially more for tuna in the pouch format. The mean premium was
33.4%, with a median of 24%. Moreover, the mean premium was significantly different from zero (t = 3.96, p < .01). Respondents were also asked to list the primary benefit of the pouch format. The modal response, provided by 19 subjects, was the ability to open the pouch without a can opener. We take these findings as evidence that consumption hassle is an important component of the overall consumption experience.

For the second survey, subjects were asked to describe their willingness to pay for a hypothetical (but plausible) service that delivered subs from a popular, local sub store. The subs would be delivered directly to them after their 11:00 – 12:15 class located on the 5th floor of the building. Students could choose their favorite sub at menu prices but would have to pay a fee for delivery. Students could also order a drink via this service or choose to pick up a drink from a student-run concessions shop on the main floor of the building. Naturally, the delivery service would charge a fee for this service also, resulting in a price higher than the concessions shop price. Finally, we imposed a time constraint on respondents by stating that they had an important Undergraduate Council Meeting at 12:30 in the same room.

In this context, we asked subjects the maximum they were willing to pay for the two services. We expected that those respondents who were willing to pay a high price for the main delivery service would also be the ones willing to pay a high price to avoid the hassle of running downstairs to purchase the drink. As expected, OLS regression supports a linear relationship. Specifically, the willingness to pay for the main delivery service predicts the effective cost of the consumption hassle (R^2 = .19, p < .01). This empirical finding provides support for our specified association between valuation and the effective cost of consumption hassle (v(θ) and α θ)).
SUMMARY AND DISCUSSION

Our research proposes a new explanation for the phenomena of quantity surcharges. Specifically, our analytical work reveals that consumption hassle can explain the puzzle of surcharges found in some product categories. More strikingly, unlike the previous research, our work can accommodate both surcharges and discounts within the same product category by explicitly recognizing variations in consumption hassle.

In our analytical work, we first examine a basic model that incorporates diminishing marginal utility with respect to quantity but excludes consumption hassle. This benchmark model yields quantity discounts as the optimal pricing schedule. We then introduce the notion of consumption hassle and show that the optimal pricing schedule now includes surcharges. We also characterize prices in the region where surcharges are optimal. An attractive feature of our analysis is that both the quantity as well as the number of packs purchased emerges endogenously within our analysis. That is, the pricing structure influences the consumer’s quantity choice as well as the manner in which that quantity is purchased.

Having provided a summary, we now say a few words regarding the broader applicability of research.

➤ The essential “force” that we propose need not be restricted to product categories where the large pack is twice the small pack. Similarly, our model “force” need not be restricted to product categories that exhibit surcharges. Consider, for example, the case of paper towels. At retail,
this product appears in a single pack as well as an 8-pack. Consumers considering the purchase of the 8-pack find some value to buying the bundled 8-pack instead of eight single packs – there is consumption hassle associated with handling eight single packs. Accordingly, our model force suggests a surcharge. However, the model forces identified by Gerstner and Hess (1987) may very well suggest a discount. If their forces dominate, the resulting pricing schedule reflects quantity discounts even though the force that we propose was present.

- Our theory also has applicability to the policy domain. In this regard, we note there have been calls in the literature to employ regulation. In discussing quantity surcharges, Gupta and Rominger (1996, p. 1310), for example, argue that “it is imperative that government, representing the interests of society, act to reduce the incidence of this deceptive practice.” They further suggest that “concerned agencies examine this issue seriously and institute appropriate regulation.” Our work offers a sharply different perspective. Our research suggests that surcharges can exist even under full-information about prices and consumer utility maximization. Thus, it is premature to invoke regulation to curb this practice.

Undoubtedly, the determination of pricing schedules is a complex phenomenon and there are many forces likely to be at play. We contribute to the rich literature in this area by proposing an additional “force” that may impact the determination of pricing
schedules. Clearly, there is a need for empirical work that explicitly examines the impact of all these forces on the optimal pricing schedule. This agenda we leave for future work.
REFERENCES


Appendix A

Here, we rule out the following market structure {Buy Nothing, Buy a single small pack, Buy two small packs, Buy a Small pack, Buy a large pack} or {φ,1,1+1,1,2}

Proof:

We prove this by contradiction. Suppose the assertion is true. Then, we would need the following to occur: U₁ should intersect U₁⁺₁ twice before U₂ intersects U₁⁺₁, and only then should U₂ intersect U₁. Thus, we need the following to occur:

- IC₁₁⁺₁ < IC₂₁⁺₁ and
- IC₂₁ > IC₂₁⁺₁

for otherwise we would be back with either {φ,1,2,} or {φ,1,1+1,2}

Now, IC₂₁⁺₁ = \(\theta = \left(\frac{p-2p}{\alpha}\right)\). IC₁₁⁺₁ has two roots. We need (as per the condition A₁) the smaller root of IC₁₁⁺₁ to be less than \((P-2p)/\alpha\).

Now the point yielding IC₁₁⁺₁ requires equality of U₁⁺₁ and U₁. Thus,

\[\theta^{1/2} - \alpha\theta - p = K\theta^{1/2} - 2\alpha\theta - 2p\]

\[\Rightarrow (K-1)\theta^{1/2} - \alpha\theta - p = 0\]

The above equation has two roots and the smaller root is

\[\frac{1}{2\alpha^2} \left((K-1)^2 - 2ap - (K-1)\sqrt{(K-1)^2 - 4ap}\right)\]. As per first part of condition A₁, we need:

\[\frac{1}{2\alpha^2} \left((K-1)^2 - 2ap -(K-1)\sqrt{(K-1)^2 - 4ap}\right) < \left(\frac{p-2p}{\alpha}\right)\]. This yields,

\[(K-1)^2 - 2ap - (K-1)\sqrt{(K-1)^2 - 4ap} < 2\alpha(P-2p)\]

\[\Rightarrow (K-1)^2 - 2\alpha(P-p) < (K-1)\sqrt{(K-1)^2 - 4ap}\] Squaring, we get:

\[(K-1)^4 + 4\alpha^2 p^2 + 4\alpha^2 p^2 - 8\alpha^2 pP - 4\alpha P(K-1)^2 + 4\alpha p(K-1)^2 < (K-1)^4 - 4\alpha p(K-1)^2\]

Cancelling like terms and simplifying, we get:

\[\left(\frac{P-2p}{\alpha}\right) > \left(\frac{P-p}{K-1}\right)^2\]
In the last inequality, the term on the right is $IC_{2,1}$ and the term on the left is $IC_{2,1+1}$. Therefore, the last inequality contradicts the second part of A1.
Appendix B

Proposition 2a:

Here, we look at the three segment solution. First, we write down the Individual Rationality and Incentive Compatibility conditions in a market where the supplier surcharges the larger pack. These are:

\[ U_1 = \theta^{\frac{1}{3}} - \alpha \theta - p \geq 0 \]  \hspace{1cm} \text{IR}_1

\[ U_2 = K \cdot \theta^{\frac{1}{3}} - \alpha \theta - P \geq 0 \]  \hspace{1cm} \text{IR}_2

\[ U_{1+1} = K \cdot \theta^{\frac{1}{3}} - 2 \alpha \theta - 2p \geq 0 \]  \hspace{1cm} \text{IR}_{1+1}

\[ \left[K-1\right]^{\frac{1}{2}} - \alpha \theta - p \geq 0 \]  \hspace{1cm} \text{IC}_{1+1,1}

\[ \alpha \theta \geq P - 2p \]  \hspace{1cm} \text{IC}_{2,1+1}

Looking at IR$_1$, we see that it is satisfied if

\[
\frac{1 - 2\alpha p - \sqrt{1 - 4\alpha p}}{2\alpha^2} \leq \theta \leq \frac{1 - 2\alpha p + \sqrt{1 - 4\alpha p}}{2\alpha^2}
\]

B1

We now turn to IC$_{1+1,1}$. This is satisfied for

\[
\frac{(K-1)^2 - 2\alpha p - (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2} \leq \theta \leq \frac{(K-1)^2 - 2\alpha p + (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2}
\]

B2

In B1 and B2, we need \(4\alpha p \leq (K - 1)^2\) for real solutions.
Taking the smaller roots of $B_1$ and $B_2$ yield the region where the market for 1 and 1+1 begin, respectively. The beginning of the market for 2 is obtained by looking at $IC_{2,1+1}$ above and is $\theta \geq \frac{P - 2p}{\alpha}$

Thus, we obtain:

$$M_1 = \frac{(K-1)^2 - 2\alpha p - (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2} - \left(1 - 2\alpha p - \frac{\sqrt{1 - 4\alpha p}}{2\alpha^2}\right)$$

$$M_{1+1} = \frac{P - 2p}{\alpha} - \left[\frac{(K-1)^2 - 2\alpha p - (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2}\right]$$

Thus, we get:

$$M_2 = 1 - \left(\frac{P - 2p}{\alpha}\right)$$

And get the profit function as:

$$\pi_{s3} = (M_1)(p - c) + (M_{1+1})(2(p - c)) + (M_2)(P - 2c) =$$

$$\left\{\frac{(K-1)^2 - 2\alpha p - (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2} - \left(1 - 2\alpha p - \frac{\sqrt{1 - 4\alpha p}}{2\alpha^2}\right)\right\}(p - c) +$$

$$\left\{\frac{P - 2p}{\alpha} - \left[\frac{(K-1)^2 - 2\alpha p - (K-1)\sqrt{(K-1)^2 - 4\alpha p}}{2\alpha^2}\right]\right\}2(p - c) +$$

$$\left\{1 - \left(\frac{P - 2p}{\alpha}\right)\right\}(P - 2c)$$

We now need to choose appropriate $p$ and $P$ so that the above is maximized. Differentiating $B_3$ with respect to $P$ and setting the derivative to zero and simplifying, we get (Proposition 2a, (i)):

$$\frac{\partial \pi_{s3}}{\partial P} = 1 - 2\left(\frac{P - 2p}{\alpha}\right) = 0 \text{ or } P = 2p + (1/2)\alpha$$

Now differentiating $B_3$ with respect to $p$, simplifying using $B4$ and setting the derivative to zero, we get:
B5

Squaring and simplifying the above yields an equation of the sixth degree in p and is not solvable in closed form.

Second order conditions: It can be shown that at the optimum,

\[ \frac{\partial^2 \Pi_{S3}}{\partial P^2} = -2, \quad \frac{\partial^2 \Pi_{S3}}{\partial P \partial p} = \frac{\partial^2 \Pi_{S3}}{\partial p \partial P} = \left[ \frac{4}{\alpha} \right] \]

\[ \frac{\partial^4 \Pi_{S3}}{\partial p^2} = -\left[ \frac{4}{\alpha} + \frac{2(K-1)}{\alpha(1-4\alpha)} \right] \left( (K-1)^2 - (c\alpha + 3\alpha p) \right) + \frac{2(1 - (c\alpha + 3\alpha p))}{(1-4\alpha)^{3/2}} \]

Using the above, it can be shown that all the second order conditions for a maximum, namely \( \frac{\partial^2 \pi_{S3}}{\partial P^2} < 0, \frac{\partial^2 \pi_{S3}}{\partial p^2} < 0, \frac{\partial^2 \pi_{S3}}{\partial P \partial p} > \frac{\partial^2 \pi_{S3}}{\partial p \partial P} \) are satisfied.

Comparative Statics (Proposition 2a, (ii)):

To see how the prices vary with K and α, we consider \( \frac{\partial p}{\partial K} \), \( \frac{\partial P}{\partial K} \), \( \frac{\partial p}{\partial \alpha} \) and \( \frac{\partial P}{\partial \alpha} \). From B4, we notice that \( \frac{\partial P}{\partial K} = \frac{\partial p}{\partial K} \) as P depends on K only via p. Thus, we need to compute \( \frac{\partial p}{\partial K} \). To do this, we implicitly differentiate the first order condition B5 with respect to K to obtain:

\[ -2(p-c) \left( \frac{1}{(K-1)^{3/2}} + \frac{1}{(1-4\alpha)^{3/2}} \right) \frac{1}{\alpha} \left( \frac{1}{\sqrt{1-4\alpha}} + \frac{K-1}{\sqrt{(K-1)^2 - 4\alpha}} \right) \frac{1}{\sqrt{1-4\alpha}} - \frac{1}{\alpha} \left( \frac{1}{\sqrt{1-4\alpha}} + \frac{K-1}{\sqrt{(K-1)^2 - 4\alpha}} \right) \frac{1}{\sqrt{1-4\alpha}} \]

\[ -\left[ \frac{1}{\alpha} \left( \frac{1}{\sqrt{1-4\alpha}} + \frac{K-1}{\sqrt{(K-1)^2 - 4\alpha}} \right) - 2 \right] \frac{\partial p}{\partial K} + \frac{1}{2\alpha^2} \left( \frac{2(K-1)^2 - 4\alpha p}{\sqrt{(K-1)^2 - 4\alpha}} - 2(K-1) \right) + \frac{(K-1)(p-c)}{\alpha((K-1)^2 - 4\alpha)^{3/2}} = 0 \]
In the above expression, it can be shown that each coefficient of the term \( \frac{\partial p}{\partial K} \) is less than zero. It can also be shown that the remaining terms are positive. This yields us an expression of the form

\[
\left( \text{Negative Quantity} \right) \left( \frac{\partial p}{\partial K} \right) + \left( \text{Positive Quantity} \right) = 0
\]

This gives \( \frac{\partial p}{\partial K} > 0 \). This also implies \( \frac{\partial P}{\partial K} > 0 \). In other words, price levels increase with \( K \).

Similarly, it can be shown that \( \frac{\partial p}{\partial \alpha} \) and \( \frac{\partial P}{\partial \alpha} \) are negative and thus, price levels decrease with \( \alpha \).
Proof of Proposition 2b:

Here, we focus on the two segment solution. In this case, we first write down the individual rationality and the incentive compatibility conditions. Thus, we first have:

\[ U_1 = \theta^{\frac{1}{2}} - \alpha \theta - p \geq 0 \]  \quad \text{IR}_1

\[ U_2 = K \cdot \theta^{\frac{1}{2}} - \alpha \theta - P \geq 0 \]  \quad \text{IR}_2

\[ \Rightarrow \theta \geq \left( \frac{P - p}{K - 1} \right)^2 \]  \quad \text{IC}_{2,1}

The market corresponds to \{\phi, 1, 2\} where the segment \phi buys nothing, segment 1 buys the single unit pack and 2 buys the two unit pack. Segment 1 comprises the consumers for whom IR_1 is satisfied but IC_{2,1} is not. Segment 2 has IC_{2,1} satisfied. We now calculate the size of the segments.

To calculate the width of 1, we solve IR_1 to get \( \theta^* \) (the point of indifference between purchase and non-purchase of the single unit pack. This yields:

\[ \theta^* = \left( \frac{1 - 2 \alpha p - \sqrt{1 - 4 \alpha p}}{2 \alpha^2} \right) \]. We take the smaller root as this is the minimum positive value of \( \theta \) for which \( U_1 \geq 0 \). Thus, the number of consumers purchasing the small unit pack can be calculated using \( \theta^* \) and IC_{2,1}. We thus have:

\[ M_1 = \left( \frac{P - p}{K - 1} \right)^2 - \left( \frac{1 - 2 \alpha p - \sqrt{1 - 4 \alpha p}}{2 \alpha^2} \right) \]

\[ M_2 = 1 - \left( \frac{P - p}{K - 1} \right)^2 \quad \text{B6} \]

The profit for the firm is thus:

\[ \Pi_2 = (p - c)(M_1) + (P - 2c)(M_2) = \left( \left( \frac{P - p}{K - 1} \right)^2 - \left( \frac{1 - 2 \alpha p - \sqrt{1 - 4 \alpha p}}{2 \alpha^2} \right) \right)(p - c) + \left( 1 - \left( \frac{P - p}{K - 1} \right)^2 \right)(P - 2c) \quad \text{B7} \]

Differentiating the above with respect to \( P \) and \( p \), we get:

\[ \frac{\partial \Pi_2}{\partial P} = 1 - 3 \left( \frac{P - p}{K - 1} \right)^2 + 2c \left( \frac{P - p}{(K - 1)^2} \right) = 0 \quad \text{B8} \]
Using B8 in the above expression, we get:

\[
\frac{\partial \Pi_2}{\partial p} = 1 + \left( \frac{p-c}{\alpha} \right) + \left( \frac{1-6\alpha p + 2ac - (1-2\alpha p)\sqrt{1-4\alpha p}}{2\alpha^2 \sqrt{1-4\alpha p}} \right)
\]

B10

The above equation is cubic in \( p \) and thus can be solved in closed form for \( p \) in terms of \( \alpha \) and \( c \) to obtain (Proposition 2b (ii))

\[
1 + \left( \frac{p-c}{\alpha} \right) + \left( \frac{1-6\alpha p + 2ac - (1-2\alpha p)\sqrt{1-4\alpha p}}{2\alpha^2 \sqrt{1-4\alpha p}} \right) = 0.
\]

B11

Also, in B8, we can solve for the quantity \( P-p \) and get:

\[
P - p = \frac{1}{3} \left( c \pm \sqrt{c^2 + 3(K-1)^2} \right). \quad \text{Here, we can reject the negative root as this would imply} \quad p > P \quad \text{and thus we are left with (Proposition 2b, (i))}
\]

\[
P - p = \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right)
\]

B12

Second order conditions:

Evaluating the second derivatives and simplifying, we get:

\[
\frac{\partial^2 \Pi_2}{\partial P^2} = -\frac{2}{(K-1)^2} (3(P-p)-c) < 0 \quad \text{as from equation B12, } 3(P-p) > c
\]

\[
\frac{\partial^2 \Pi_2}{\partial p^2} = -\frac{1}{\alpha(1-4\alpha p)^{3/2}} \left[ 2 - \left( 1 - \frac{1-4\alpha p}{1-4\alpha p} + c\alpha + 3\alpha\alpha \right) \right] - \frac{2}{(K-1)^2} (3(P-p)-c) < 0
\]

\[
\frac{\partial^2 \Pi_2}{\partial P \partial p} = \frac{\partial^2 \Pi_2}{\partial p \partial P} = \frac{2}{(K-1)^2} (3(P-p)-c) > 0
\]

Using the above, we can easily verify that \( \frac{\partial^2 \Pi_2}{\partial P^2} \left( \frac{\partial^2 \Pi_2}{\partial p^2} \right) > \left( \frac{\partial^2 \Pi_2}{\partial P \partial p} \right) \left( \frac{\partial^2 \Pi_2}{\partial P \partial p} \right) \) and thus, the second order conditions are satisfied.

Comparative statics (Proposition 2b, (iv))
From B9, we can see that \( p \) is independent of \( K \) and from B12, we can see that \( P \) is increasing in \( K \) and thus \( \partial P / \partial K > 0 \) and \( \partial p / \partial K = 0 \).

For change with \( \alpha \), we first see that \( (\partial P / \partial \alpha) = (\partial p / \partial \alpha) \) since from condition B11, it is evident that dependence of \( P \) on \( \alpha \) is only through \( p \). To evaluate these, we consider the first order conditions:

\[
\frac{\partial \Pi_2}{\partial P} = 0, \quad \frac{\partial \Pi_2}{\partial p} = 0.
\]

Totally differentiating the above keeping other parameters (\( c \) and \( K \)) constant, we get:

\[
\frac{\partial^2 \Pi_2}{\partial P^2} dP + \frac{\partial^2 \Pi_2}{\partial \alpha \partial p} d\alpha + \frac{\partial^2 \Pi_2}{\partial p \partial \alpha} dp = 0 &
\]

\[
\frac{\partial^2 \Pi_2}{\partial p^2} dp + \frac{\partial^2 \Pi_2}{\partial \alpha \partial p} d\alpha + \frac{\partial^2 \Pi_2}{\partial P \partial \alpha} dP = 0
\]

Treating the above as simultaneous equations, we obtain:

\[
\begin{pmatrix}
\frac{dP}{d\alpha} \\
\frac{dp}{d\alpha} \\
\frac{dP}{dp}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial^2 \Pi_2}{\partial P^2} & \frac{\partial^2 \Pi_2}{\partial \alpha \partial p} & \frac{\partial^2 \Pi_2}{\partial p \partial \alpha} \\
\frac{\partial^2 \Pi_2}{\partial \alpha \partial p} & \frac{\partial^2 \Pi_2}{\partial \alpha^2} & \frac{\partial^2 \Pi_2}{\partial \alpha \partial p} \\
\frac{\partial^2 \Pi_2}{\partial p \partial \alpha} & \frac{\partial^2 \Pi_2}{\partial \alpha \partial p} & \frac{\partial^2 \Pi_2}{\partial \alpha^2}
\end{pmatrix}^{-1} \begin{pmatrix}
\frac{\partial^2 \Pi_2}{\partial P}\frac{\partial \Pi_2}{\partial \alpha} \\
\frac{\partial^2 \Pi_2}{\partial \alpha \partial P} \\
\frac{\partial^2 \Pi_2}{\partial \alpha \partial \alpha}
\end{pmatrix}
\]

Inverting the matrix, we get:

\[
\frac{dp}{d\alpha} = -\frac{A}{B}
\]

B13

Where \( A= \text{numerator} \) and \( B = \left( \frac{\partial^2 \Pi_2}{\partial P^2} \right) \left( \frac{\partial^2 \Pi_2}{\partial \alpha \partial \alpha} \right) - \left( \frac{\partial^2 \Pi_2}{\partial p \partial \alpha} \right) \left( \frac{\partial^2 \Pi_2}{\partial P \partial \alpha} \right) > 0 \) from second order conditions.

We also know that \( \frac{\partial^2 \Pi_2}{\partial \alpha \partial P} = \frac{\partial}{\partial \alpha} \left( \frac{\partial \Pi_2}{\partial P} \right) = 0 \) since \( \frac{\partial \Pi_2}{\partial P} \) does not depend on \( \alpha \) from condition B8. Since the denominator \( B > 0 \), the sign of \( (\partial p / \partial \alpha) \) is determined by the numerator. Further, since \( \frac{\partial^2 \Pi_2}{\partial \alpha \partial \alpha} = 0 \), the second term in the numerator of B13 vanishes.
Also, since \( \frac{\partial^2 \Pi}{\partial p^2} < 0 \) from the second order conditions, looking at equation B13, we have, \( \text{sign } (dp/d\alpha) = \text{sign}(\partial^2 \Pi/\partial \alpha \partial p) \). Evaluating \( (\partial^2 \Pi/\partial \alpha \partial p) \) using equation B9, we get:

\[
\frac{\partial^2 \Pi}{\partial \alpha \partial p} = \frac{1}{\alpha^2} \left( \frac{1}{\alpha} - 2p + c + \frac{6p}{\sqrt{1-4ap}} - \frac{2}{\alpha \sqrt{1-4ap}} \right)
\]

\[
= \frac{1}{\alpha^2} \left( \frac{\sqrt{1-4ap}(1-2ap + c\alpha) - 2(1-3ap)}{\alpha \sqrt{1-4ap}} \right)
\]

\[
< \frac{1}{\alpha^2} \left( \frac{(1-2ap)(1-2ap + c\alpha) - 2(1-3ap)}{\alpha \sqrt{1-4ap}} \right) \text{ as } (1-2ap) > \sqrt{1-4ap}
\]

Simplifying, we get:

\[
\frac{\partial^2 \Pi}{\partial \alpha \partial p} < \frac{1}{\alpha^2} \left( \frac{4p^* \alpha^2 - 2c\alpha^2 + 2p\alpha + c\alpha - 1}{\alpha \sqrt{1-4ap}} \right) < \frac{1}{\alpha^2} \left( \frac{p\alpha(4p\alpha) - 2c\alpha^2 + 2p\alpha + p\alpha - 1}{\alpha \sqrt{1-4ap}} \right)
\]

\[
= \frac{1}{\alpha^2} \left( \frac{-2c\alpha^2 + 4p\alpha - 1}{\alpha \sqrt{1-4ap}} \right) < 0 \text{ since } 1 > 4ap \text{ and } p > c
\]

Thus, \( \frac{\partial^2 \Pi}{\partial \alpha \partial p} < 0 \) and consequently \( \frac{\partial p}{\partial \alpha} < 0 \) and \( \frac{\partial P}{\partial \alpha} < 0 \) (as \( P \) depends on \( \alpha \) only via \( p \)). In other words, price levels decrease with increasing \( \alpha \).

**Condition for surcharges:** Condition B12 states, \( P - p = \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right) \). Thus, we can see that if \( p < \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right) \), we would have \( P - p > p \) and thus, \( P > 2p \) and we would have quantity surcharges. On the other hand, if \( p > \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right) \), we would have \( P < 2p \) and we would have quantity discounts.

Labeling \( p^* = \frac{1}{3} \left( c + \sqrt{c^2 + 3(K-1)^2} \right) \), and substituting this value in \( (\partial \Pi/\partial p) \), if we find \( \frac{\partial \Pi}{\partial p} \bigg|_{p=p^*} < 0 \), it would imply that the equilibrium price for the small pack is less than \( p^* \) and the price schedule would involve surcharges. Thus, substituting \( p=p^* \) in the expression for \( \partial \Pi/\partial p \), the condition \( \frac{\partial \Pi}{\partial p} \bigg|_{p=p^*} < 0 \) simplifies to (after some manipulation):
\[
\frac{3\sqrt{3} \left[ 1 - 2\alpha \sqrt{c^2 + 3(K - 1)^2} \right]}{\sqrt{3 - 4\alpha \sqrt{c^2 + 3(K - 1)^2} - 4\alpha}} < 3 + 2\alpha - \left( 6\alpha^2 + 4\alpha \sqrt{c^2 + 3(K - 1)^2} \right)
\]

B14 is now the necessary and sufficient condition for surcharges in the market with two segments (Proposition 2b, (iii)).
Figure 1. Utilities and Market Partitions
Legend:  
D2: Market is $[\phi, 1, 2]$, Pricing is Discounts  
S2: Market is $[\phi, 1, 2]$, Pricing is Surcharges  
S3: Market is $[\phi, 1, 1+1, 2]$, Pricing is Surcharges  

Figure 2: Market Structure and Pricing across Parameter Space